Speed of Sound and Fundamental Equation of State for *n*-Hydrogen

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Task 2.2: Define a procedure to calculate *C**

2.2.2 Measure speed of sound up to 100 MPa
2.2.3 Develop a new fundamental equation of state

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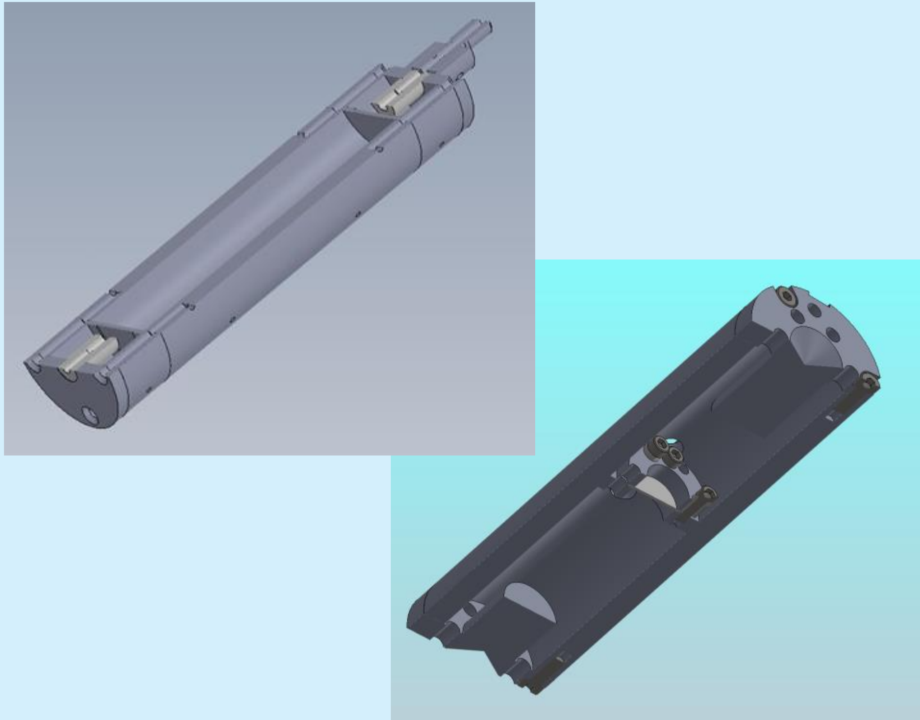
Experimental measurements



- Two sensors:
  - a) Cylindrical resonator up to 10 MPa
  - b) Dual-path pulse-echo 15 to 100 MPa
- Temperatures 273 K to 323 K
- Pathlength calibration with He and N_2
  - – relative uncertainty ≈ 0.01%
- Overall relative uncertainty ≈ 0.03% ($k = 2$)

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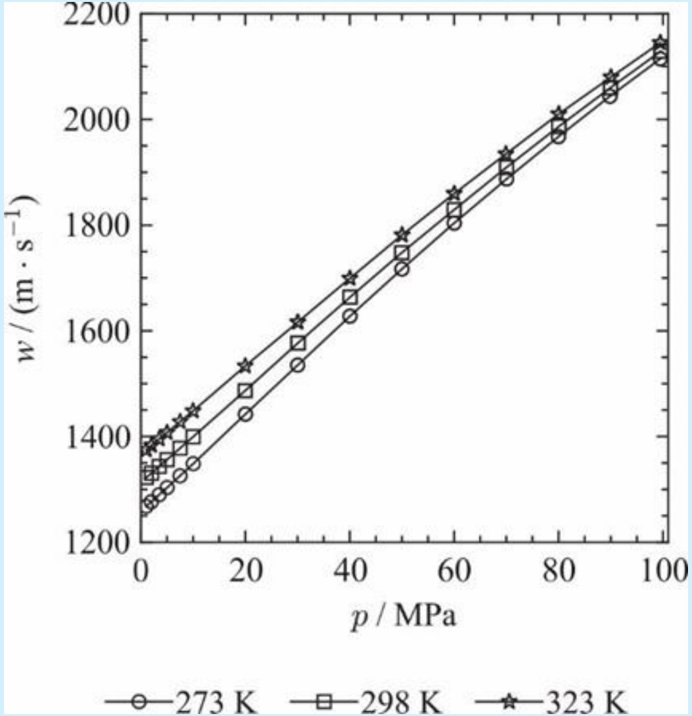
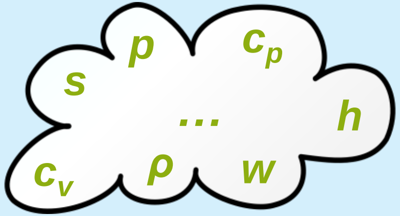


Helmholtz-Energy Equation of State

$$\frac{a(T,\rho)}{RT} = \alpha(\tau,\delta) = \alpha^{\mathrm{o}}(\tau,\delta) + \alpha^{\mathrm{r}}(\tau,\delta)$$



Derived from the isobaric heat capacity of the ideal gas

Empirical approach to account for intermolecular forces

- **Range of validity:**
  140 < T/K < 370 and p_{max} = 100 MPa
- Special requirements for fast and accurate calculations
  - As short and simple as possible
  - Limited to integer exponents
  - Reproduce available experimental data within measurement uncertainties

- Available experimental data
  - Homogeneous density $p\rho T$ (literature data)
  - Thermal virial coeffcients B and C (literature data)
  - Speed of sound pwT (MetHyInfra)

EOS:

$$\alpha^{\mathrm{r}}(\tau,\delta) = \sum_{i=1}^{11} n_i \tau^{t_i} \delta^{d_i}$$

$t_i, d_i \in \mathbb{Z}$

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Helmholtz-Energy Equation of State

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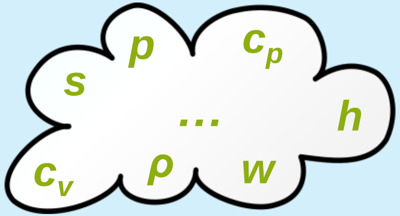


Combination of ← T (τ) and ρ (δ) derivatives

$$\frac{a(T,\rho)}{RT} = \alpha(\tau,\delta) = \alpha^{\mathrm{o}}(\tau,\delta) + \alpha^{\mathrm{r}}(\tau,\delta)$$

Pressure

$$\frac{p}{\rho RT} = 1 + \delta\alpha_\delta^{\mathrm{r}}$$

Slope of isotherm

(main contribution to isothermal compressibility)

$$\left(\frac{\partial p}{\partial \rho}\right)_T = RT\left(1 + 2\delta\alpha_\delta^{\mathrm{r}} + \delta^2\alpha_{\delta\delta}^{\mathrm{r}}\right)$$

Speed of Sound

$$\frac{w^2}{RT} = 1 + 2\delta\alpha_\delta^{\mathrm{r}} + \delta^2\alpha_{\delta\delta}^{\mathrm{r}} - \frac{1+\delta\alpha_\delta^{\mathrm{r}}-\delta\tau\alpha_{\delta\tau}^{\mathrm{r}}}{\tau^2(\alpha_{\tau\tau}^{\mathrm{o}}+\alpha_{\tau\tau}^{\mathrm{r}})}$$

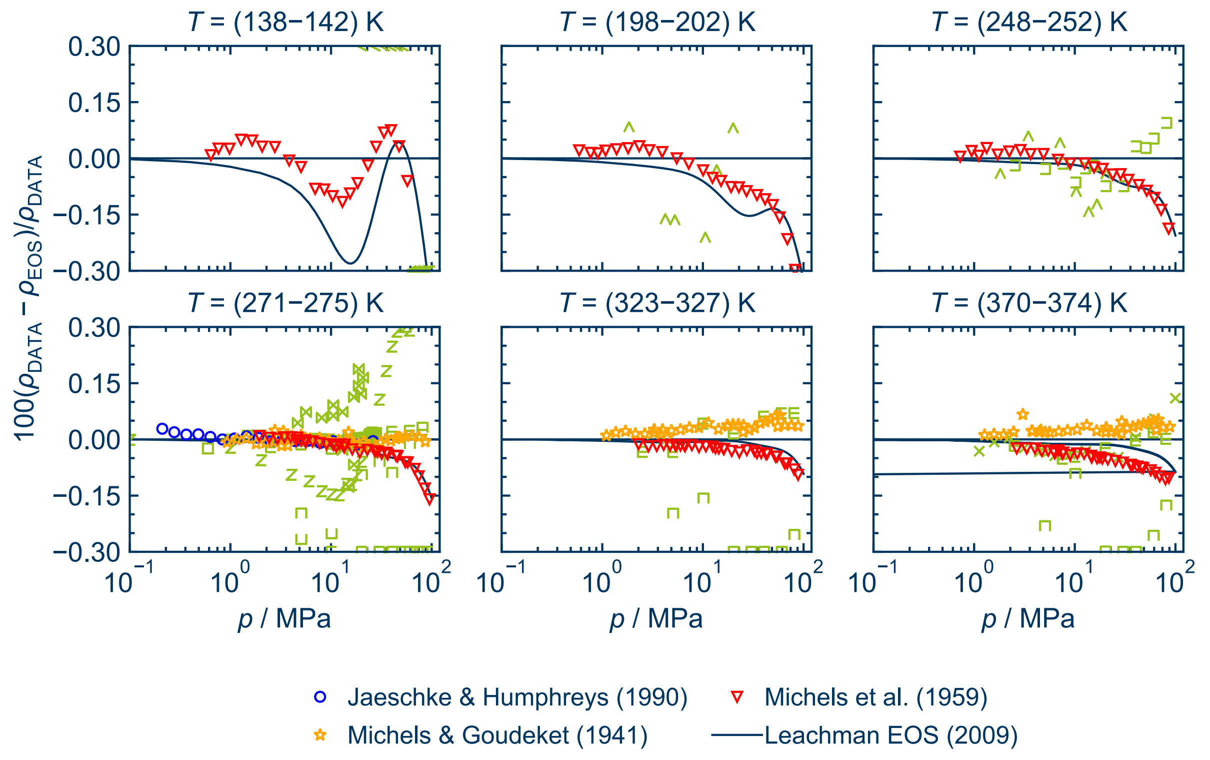
If all thermodynamic properties can be calculated from Helmholtz derivatives, fitting works vice versa:

all properties can be used to adjust the EOS parameters

Same information from two different properties

Comparison with density data

(wide temperature range)



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Comparison with density and sound speed

(273 K to 323 K)

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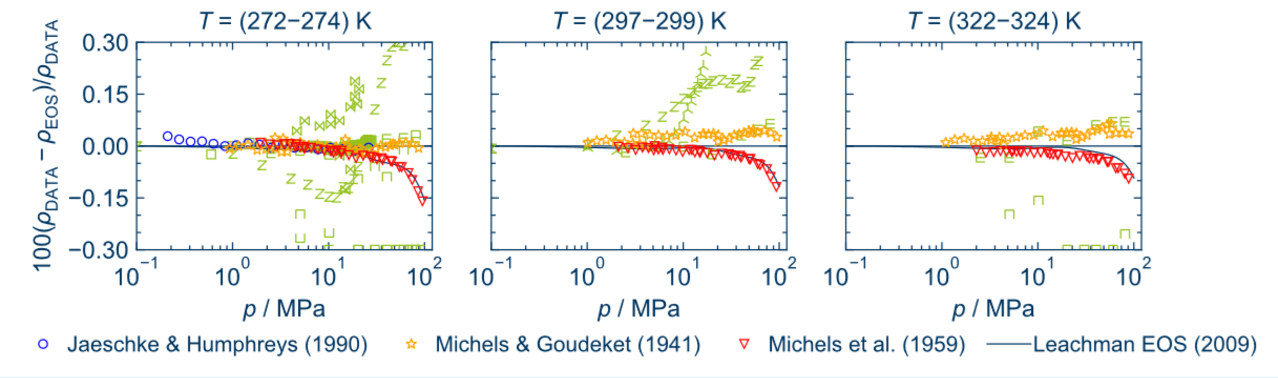
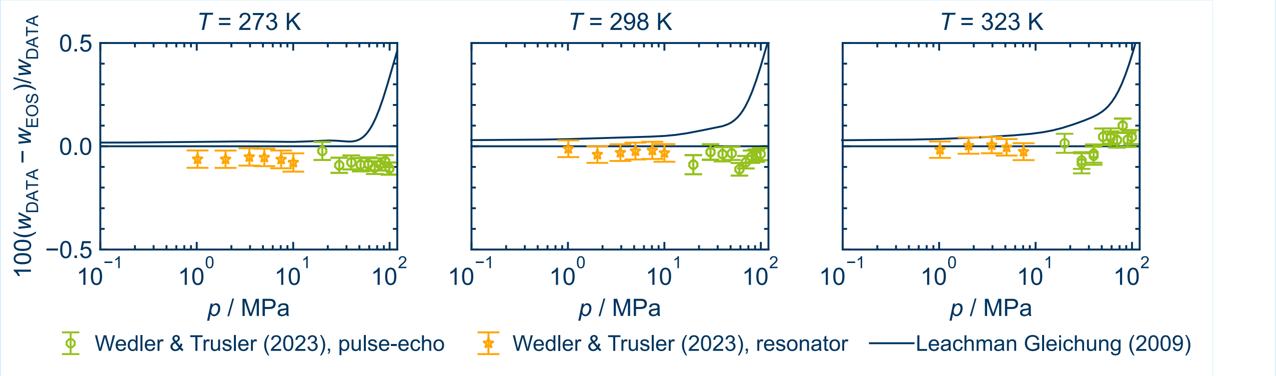


EOS Uncertainty for Density:
270 to 350 K, p < 30 MPa, 0.03%
140 to 370 K, p < 100 MPa, 0.3%



EOS Uncertainty for SoS:
273 to 323 K, p < 100 MPa, 0.08%

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